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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 2, 2017/2018 SESSION

**BMS2024 - ADVANCED MANAGERIAL STATISTICS**  
(All Sections / Groups)

17 MARCH 2018  
2.30 p.m. – 4.30 p.m.  
(2 Hours)

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### INSTRUCTIONS TO STUDENTS

1. This question paper consists of 12 pages **excluding** the cover page.
2. This question paper consists of **FOUR** structured questions. Attempt **ALL** questions.
3. Students are allowed to use non-programmable scientific calculators with no restrictions.
4. A formulae list and statistical tables are attached at the end of the question paper.

**QUESTION 1**

- a) Suppose a pickup and delivery company states that its packages arrive within two days or less on average. You want to find out whether the actual average delivery time is longer than the company's claim. You conduct a hypothesis test.

(i) Set up the null and alternative hypotheses. [3 marks]

(ii) Suppose you conclude wrongly that the company's statement about average delivery time is within two days. What type of error is being committed and what is the impact of that error? [3 marks]

(iii) Suppose you conclude wrongly that the delivery company's average time to deliver packages is in fact longer than two days. What type of error did you commit and what is the impact of this error? [3 marks]

(iv) Which error is worse from the company's standpoint; a Type I or a Type II error? Why? [2 marks]

(v) Which error is worse from a consumer standpoint; a Type I or a Type II error? Why? [2 marks]

- b) Determine a Type II error for the following test of hypothesis, given that  $\mu_T = 48$ ,  $\sigma = 10$ ,  $n = 40$ . Use  $\alpha = 0.05$ .

$$H_0: \mu = 50$$

$$H_1: \mu < 50$$

Compute and explain the power of the test. [12 marks]

[Total: 25 Marks]

Continued...

**QUESTION 2**

- a) Differentiate between Kruskal-Wallis test and the One-Way Analysis of Variance in terms of their assumptions and the circumstances under which each should be applied. [4 marks]
- b) Explain the similarity and difference between the Wilcoxon rank sum technique and Kruskal-Wallis test. [4 marks]
- c) In an experiment to determine which of three different systems is preferable, the efficiency rate is measured. The data, after coding, are given in the table below. At the 0.05 level of significance, can we conclude that the median efficiency rates for the three systems are the same? Show all steps and workings. [17 marks]

Efficiency Rates		
System		
1	2	3
24.0	23.2	18.4
16.7	19.8	19.1
22.8	18.1	17.3
19.8	17.6	17.3
18.9	20.2	19.7
	17.8	18.9
		18.8
		19.3

[Total: 25 Marks]

Continued...

**QUESTION 3**

The business problem facing a real estate developer involves predicting air-conditioner consumption( in kilowatt Hour, kWh) in a particular house. The independent variables considered are atmosphere temperature ( $^{\circ}\text{F}$ ),  $X_1$ ; the amount of roof insulation (inches),  $X_2$  and the number of storeys in a house,  $X_3$ . Data are collected from a sample of 15 houses. The data are analysed and the summary output of the analysis is shown below:

<i>Regression Statistics</i>					
Multiple R	0.9942				
R Square	0.9884				
Adjusted R Square	0.9853				
Standard Error	15.7489				
Observations	15				

<i>ANOVA</i>					
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	3	233406.9094	77802.3031	313.6822	0.0000
Residual	11	2728.3200	248.0291		
Total	14	236135.2293			

	<i>Coefficients</i>	<i>Std Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	592.5401	14.3370	41.3295	0.0000
$X_1$	-5.5251	0.2044	-27.0267	0.0000
$X_2$	-21.3761	1.4480	-14.7623	0.0000
$X_3$	38.9727	8.3584	4.6627	0.0007

- State the multiple linear regression equation for the above data. [5 marks]
- Interpret all the three regression coefficients,  $b_1$ ,  $b_2$  and  $b_3$  for the equation in (a). [6 marks]
- Compare and interpret the values of the coefficient of determination with the adjusted coefficient of determination. Explain why the values differ. [4 marks]
- At the 1 percent level of significance, test if the model is valid. Use the p-value approach. [5 marks]
- At the 1 percent level of significance, test if each independent variable have a significant contribution to the model. Use the p-value approach. [5 marks]

[Total: 25 Marks]

Continued...

**QUESTION 4**

In an experiment to determine the effect of nutrition on the attention spans of primary school students, a group of 15 students were randomly selected and assigned to each of three meal plans: no breakfast, light breakfast and full breakfast. Their attention spans (in minutes) were recorded during a morning reading period and are shown in the following table:

No breakfast	Light Breakfast	Full Breakfast
7	14	16
10	17	15
8	16	10
9	11	12
13	12	12

**Summary Output**

Groups	Count	Sum	Mean	Variance
No breakfast	5	47	9.4	5.3
Light breakfast	5	70	14	6.5
Full breakfast	5	65	13	6.0

**ANOVA**

Source of Variation	SS	df	MS	F
Among groups	58.53	2	29.265	4.933
Within groups	71.2	12	5.933	
Total	129.73	14		

- State the required conditions or assumptions for the ANOVA test to be conducted?  
[3 marks]
- What kind of ANOVA test will be appropriate for this study? What are the dependent variable and the independent variables (factors) that needs to be identified? Conduct an appropriate statistical procedure in testing the difference in the attention spans received by the three sample means? Test at  $\alpha = 0.05$ .  
[12 marks]
- Conduct the Tukey-Kramer post-hoc test to examine if there is any significant change in the attention spans received by the three sample means. (NB: The value of  $Q_u$  in the Critical Range formula is 3.77)  
[10 marks]

**[Total: 25 marks]**

**End of Paper**

## STATISTICAL FORMULAE

### A. DESCRIPTIVE STATISTICS

$\text{Sample Mean} = \bar{X} = \frac{\sum_{i=1}^n X_i}{n}$ $\text{Sample Standard Deviation} = s = \sqrt{\frac{\sum_{i=1}^n X_i^2}{n-1} - \frac{\left(\sum_{i=1}^n X_i\right)^2}{n(n-1)}}$ <p>where <math>n</math> = number of observations  <math>X_i</math> = the <math>i^{\text{th}}</math> observation of the data</p>
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### B. HYPOTHESIS TESTING

Types of Error
Type I Error = $\alpha$ = P(Rejecting $H_0$   $H_0$ is true) where, Confidence Interval = $1 - \alpha$
Type II Error = $\beta$ = P(Not Rejecting $H_0$   $H_0$ is false)

One Sample Mean Test	
$\sigma$ Known	$\sigma$ Unknown
$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$	$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$
Two Sample Mean Test	
Comparing Means for Two Independent Populations	
<p>[Standard Deviation (<math>\sigma</math>) Known]</p> $z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	<p>[Standard Deviation (<math>\sigma</math>) Not Known]</p> $t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$ <p>where <math>S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}</math></p>

Two Sample Mean Test	
Comparing Means for Two Paired Populations	
$t = \frac{(\bar{D} - \mu_D)}{S_D / \sqrt{n}}$	<p>where <math>\bar{D} = \frac{\sum_{i=1}^n D_i}{n}</math> and <math>S_D = \sqrt{\frac{\sum_{i=1}^n D_i^2}{n-1} - \frac{\left(\sum_{i=1}^n D_i\right)^2}{n(n-1)}}</math></p>

Non-Parametric Analysis	
Wilcoxon Rank Sum Test	Wilcoxon Signed Rank Sum Test
$Z = \frac{(T_1 - \mu_{T_1})}{\sigma_{T_1}} \quad \text{where}$ $\mu_{T_1} = \frac{n_1(n+1)}{2} \quad \text{and}$ $\sigma_{T_1} = \sqrt{\frac{n_1 n_2 (n+1)}{12}} \quad \text{where } n = n_1 + n_2$	$Z = \frac{(T_+ - \mu_{T_+})}{\sigma_{T_+}} \quad \text{where}$ $\mu_{T_+} = \frac{n(n+1)}{4} \quad \text{and}$ $\sigma_{T_+} = \sqrt{\frac{n(n+1)(2n+1)}{24}}$
Kruskal-Wallis Rank Test	
$H = \left[ \frac{12}{n(n+1)} \sum_{j=1}^c \frac{T_j^2}{n_j} \right] - 3(n+1) \quad \text{where the critical value is } \chi^2 \text{ with } df = c - 1$ <p>Check ranking sum: <math>\sum T_j = n(n+1)/2</math></p>	

Chi-Square Test
$\chi^2 = \sum \frac{(O - E)^2}{E}$ <p>where <math>O</math> = Frequency of Observed Values and <math>E</math> = Frequency of Expected Values</p> <p>with <math>df = c - 1</math> where <math>c</math> = number of categories</p> <p>or</p> <p>with <math>df = (r - 1)(c - 1)</math> where <math>r</math> = number of rows and <math>c</math> = number of columns</p>

### C. ANALYSIS OF VARIANCE (ANOVA)

One-Way ANOVA				
Source	Degrees of Freedom	Sum of Squares	Mean Squares	F-statistic
Among Groups	$c - 1$	SSA	$MSA = SSA/c - 1$	$MSA/MSW$
Within Groups	$n - c$	SSW	$MSW = SSW/n - c$	
Total	$n - 1$	SST		
$SST = \sum_{j=1}^c \sum_{i=1}^{n_j} (X_{ij} - \bar{\bar{X}})^2 \quad \text{or alternative formula:}$ $SSA = \sum_{j=1}^c n_j (\bar{X}_j - \bar{\bar{X}})^2 \quad \text{and } SSW = SST - SSA$ $SST = \left( \sum_{j=1}^c \sum_{i=1}^{n_j} X_{ij}^2 \right) - \frac{\left( \sum_{j=1}^c \sum_{i=1}^{n_j} X_{ij} \right)^2}{n}$ <p>where <math>n</math> = number of observations, <math>c</math> = number of groups and <math>\bar{\bar{X}}</math> = overall mean</p>				

**Tukey-Kramer Procedure**

$$\text{Critical Range} = Q_u \sqrt{\frac{MSW}{2} \left[ \frac{1}{n_i} + \frac{1}{n_j} \right]}$$

where  $Q_u$  = the upper tail critical value from a Studentized Range Distribution having  $(c)$  degrees of freedom in the numerator and  $(n - c)$  degrees of freedom in the denominator at a given level of significance

Two-Way ANOVA				
Source	Degrees of Freedom	Sum of Squares	Mean Squares	F-statistic
A	$r - 1$	SSA	$MSA = SSA/(r - 1)$	$MSA / MSE$
B	$c - 1$	SSB	$MSB = SSB/(c - 1)$	$MSB / MSE$
AB	$(r - 1)(c - 1)$	SSAB	$MSAB = SSAB/(r - 1)(c - 1)$	$MSAB / MSE$
Error	$rc(n - 1)$	SSE	$MSE = SSE/rc(n - 1)$	
Total	$n - 1$	SST		

where,

Factor A levels are represented by the rows and Factor B levels are represented by the columns and

$n$  = number of observations

$c$  = number of columns

$r$  = number of rows

$n'$  = number of replicates

$$SST = \sum_{i=1}^r \sum_{j=1}^c \sum_{k=1}^{n'} (X_{ijk} - \bar{X})^2 \quad SSA = cn' \sum_{i=1}^r (\bar{X}_i - \bar{X})^2$$

$$SSB = rn' \sum_{j=1}^c (\bar{X}_j - \bar{X})^2 \quad \text{where } \bar{X} = \text{overall mean}$$

$$SSE = (n' - 1)[S_1^2 + S_2^2 + \dots + S_k^2] \quad \text{where } S_i^2 = \text{variance of each block}$$

**D. REGRESSION ANALYSIS****Multiple Linear Regression**

Population Model:  $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_k X_k + \varepsilon$

Sample Model:  $\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_k x_k + e$

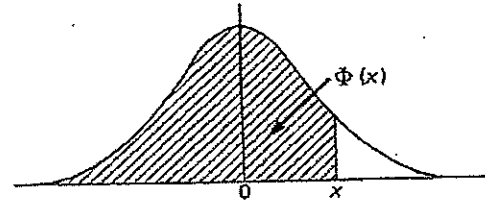
$$\text{Adjusted R-Square} = 1 - \left[ \frac{(1 - R^2)(n - 1)}{(n - p - 1)} \right] \quad \text{where } p = \text{number of independent/predictor variables}$$

ANOVA Table for Regression			
Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	$p$	SSR	$MSR = SSR/p$
Error/Residual	$n - p - 1$	SSE	$MSE = SSE/(n - p - 1)$
Total	$n - 1$	SST	
<b>Test Statistic for Significance of the Overall Regression Model</b> $F = MSR/MSE$			
<b>Test Statistic for Significance of Each Predictor Variable</b> $t_i = \frac{b_i}{S_{b_i}}$ and the critical value = $\pm t_{\alpha/2, (n-p-1)}$			

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt$ .  $\Phi(x)$  is

the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to  $x$ . When  $x < 0$  use  $\Phi(x) = 1 - \Phi(-x)$ , as the normal distribution with zero mean and unit variance is symmetric about zero.



$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$	$x$	$\Phi(x)$
0.00	0.5000	0.40	0.6554	0.80	0.7881	1.20	0.8849	1.60	0.9452	2.00	0.97725
0.01	5040	0.41	6591	0.81	7910	1.21	8869	1.61	9463	2.01	97778
0.02	5080	0.42	6628	0.82	7939	1.22	8888	1.62	9474	2.02	97831
0.03	5120	0.43	6664	0.83	7967	1.23	8907	1.63	9484	2.03	97882
0.04	5160	0.44	6700	0.84	7995	1.24	8925	1.64	9495	2.04	97932
0.05	5199	0.45	6736	0.85	8023	1.25	8944	1.65	9505	2.05	97982
0.06	5239	0.46	6772	0.86	8051	1.26	8962	1.66	9515	2.06	98030
0.07	5279	0.47	6808	0.87	8078	1.27	8980	1.67	9525	2.07	98077
0.08	5319	0.48	6844	0.88	8106	1.28	8997	1.68	9535	2.08	98124
0.09	5359	0.49	6879	0.89	8133	1.29	9015	1.69	9545	2.09	98169
0.10	5398	0.50	6915	0.90	8159	1.30	9032	1.70	9554	2.10	98214
0.11	5438	0.51	6950	0.91	8186	1.31	9049	1.71	9564	2.11	98257
0.12	5478	0.52	6985	0.92	8212	1.32	9066	1.72	9573	2.12	98300
0.13	5517	0.53	7019	0.93	8238	1.33	9082	1.73	9582	2.13	98341
0.14	5557	0.54	7054	0.94	8264	1.34	9099	1.74	9591	2.14	98382
0.15	5596	0.55	7088	0.95	8289	1.35	9115	1.75	9599	2.15	98422
0.16	5636	0.56	7123	0.96	8315	1.36	9131	1.76	9608	2.16	98461
0.17	5675	0.57	7157	0.97	8340	1.37	9147	1.77	9616	2.17	98500
0.18	5714	0.58	7190	0.98	8365	1.38	9162	1.78	9625	2.18	98537
0.19	5753	0.59	7224	0.99	8389	1.39	9177	1.79	9633	2.19	98574
0.20	5793	0.60	7257	1.00	8413	1.40	9192	1.80	9641	2.20	98610
0.21	5832	0.61	7291	1.01	8438	1.41	9207	1.81	9649	2.21	98645
0.22	5871	0.62	7324	1.02	8461	1.42	9222	1.82	9656	2.22	98679
0.23	5910	0.63	7357	1.03	8485	1.43	9236	1.83	9664	2.23	98713
0.24	5948	0.64	7389	1.04	8508	1.44	9251	1.84	9671	2.24	98745
0.25	5987	0.65	7422	1.05	8531	1.45	9265	1.85	9678	2.25	98778
0.26	6026	0.66	7454	1.06	8554	1.46	9279	1.86	9686	2.26	98809
0.27	6064	0.67	7486	1.07	8577	1.47	9292	1.87	9693	2.27	98840
0.28	6103	0.68	7517	1.08	8599	1.48	9306	1.88	9699	2.28	98870
0.29	6141	0.69	7549	1.09	8621	1.49	9319	1.89	9706	2.29	98899
0.30	6179	0.70	7580	1.10	8643	1.50	9332	1.90	9713	2.30	98928
0.31	6217	0.71	7611	1.11	8665	1.51	9345	1.91	9719	2.31	98956
0.32	6255	0.72	7642	1.12	8686	1.52	9357	1.92	9726	2.32	98983
0.33	6293	0.73	7673	1.13	8708	1.53	9370	1.93	9732	2.33	99010
0.34	6331	0.74	7704	1.14	8729	1.54	9382	1.94	9738	2.34	99036
0.35	6368	0.75	7734	1.15	8749	1.55	9394	1.95	9744	2.35	99061
0.36	6406	0.76	7764	1.16	8770	1.56	9406	1.96	9750	2.36	99086
0.37	6443	0.77	7794	1.17	8790	1.57	9418	1.97	9756	2.37	99111
0.38	6480	0.78	7823	1.18	8810	1.58	9429	1.98	9761	2.38	99134
0.39	6517	0.79	7852	1.19	8830	1.59	9441	1.99	9767	2.39	99158
0.40	6554	0.80	7881	1.20	8849	1.60	9452	2.00	9772	2.40	99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$
2.40	0.99180	2.55	0.99461	2.70	0.99653	2.85	0.99781	3.00	0.99865	3.15	0.99918
41	99202	56	99477	71	99664	86	99788	01	99869	16	99921
42	99224	57	99492	72	99674	87	99795	02	99874	17	99924
43	99245	58	99506	73	99683	88	99801	03	99878	18	99926
44	99266	59	99520	74	99693	89	99807	04	99882	19	99929
2.45	0.99286	2.60	0.99534	2.75	0.99702	2.90	0.99813	3.05	0.99886	3.20	0.99931
46	99305	61	99547	76	99711	91	99819	06	99889	21	99934
47	99324	62	99560	77	99720	92	99825	07	99893	22	99936
48	99343	63	99573	78	99728	93	99831	08	99896	23	99938
49	99361	64	99585	79	99736	94	99836	09	99900	24	99940
2.50	0.99379	2.65	0.99598	2.80	0.99744	2.95	0.99841	3.10	0.99903	3.25	0.99942
51	99396	66	99609	81	99752	96	99846	11	99906	26	99944
52	99413	67	99621	82	99760	97	99851	12	99910	27	99946
53	99430	68	99632	83	99767	98	99856	13	99913	28	99948
54	99446	69	99643	84	99774	99	99861	14	99916	29	99950
2.55	0.99461	2.70	0.99653	2.85	0.99781	3.00	0.99865	3.15	0.99918	3.30	0.99952

The critical table below gives on the left the range of values of  $z$  for which  $\Phi(z)$  takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of  $\Phi(z)$  indicated.

3.075	0.9990	3.263	0.9994	3.731	0.99990	3.916	0.99995
3.105	0.9990	3.320	0.9995	3.759	0.99991	3.976	0.99996
3.138	0.9991	3.389	0.9996	3.791	0.99992	4.055	0.99997
3.174	0.9992	3.480	0.9997	3.826	0.99993	4.173	0.99998
3.215	0.9993	3.615	0.9998	3.867	0.99994	4.417	1.00000
	0.9994		0.9999		0.99995		

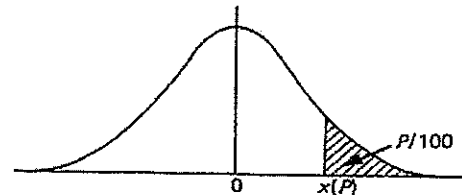
When  $z > 3.3$  the formula  $1 - \Phi(z) \approx \frac{e^{-z^2}}{z\sqrt{2\pi}} \left[ 1 - \frac{1}{z^2} + \frac{3}{z^4} - \frac{15}{z^6} + \frac{105}{z^8} \right]$  is very accurate, with relative error less than  $945/z^{10}$ .

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points  $z(P)$  defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{z(P)}^{\infty} e^{-t^2/2} dt.$$

If  $X$  is a variable, normally distributed with zero mean and unit variance,  $P/100$  is the probability that  $X \geq z(P)$ . The lower  $P$  per cent points are given by symmetry as  $-z(P)$ , and the probability that  $|X| \geq z(P)$  is  $2P/100$ .



$P$	$z(P)$	$P$	$z(P)$	$P$	$z(P)$	$P$	$z(P)$	$P$	$z(P)$	$P$	$z(P)$
50	0.0000	5.0	1.6449	3.0	1.8808	2.0	2.0537	1.0	2.3263	0.10	3.0902
45	0.1257	4.8	1.6646	2.9	1.8957	1.9	2.0749	0.9	2.3656	0.09	3.1214
40	0.2533	4.6	1.6849	2.8	1.9110	1.8	2.0969	0.8	2.4089	0.08	3.1559
35	0.3853	4.4	1.7060	2.7	1.9268	1.7	2.1201	0.7	2.4573	0.07	3.1947
30	0.5244	4.2	1.7279	2.6	1.9431	1.6	2.1444	0.6	2.5121	0.06	3.2389
25	0.6745	4.0	1.7507	2.5	1.9600	1.5	2.1701	0.5	2.5758	0.05	3.2905
20	0.8416	3.8	1.7744	2.4	1.9774	1.4	2.1973	0.4	2.6521	0.04	3.7190
15	1.0364	3.6	1.7991	2.3	1.9954	1.3	2.2262	0.3	2.7478	0.005	3.8906
10	1.2816	3.4	1.8250	2.2	2.0141	1.2	2.2571	0.2	2.8782	0.001	4.2649
5	1.6449	3.2	1.8522	2.1	2.0335	1.1	2.2904	0.1	3.0902	0.0005	4.4172

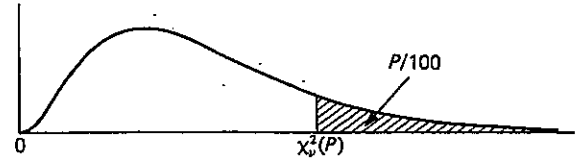
TABLE 8. PERCENTAGE POINTS OF THE  $\chi^2$ -DISTRIBUTION

This table gives percentage points  $\chi^2_\nu(P)$  defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2} \Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\nu/2-1} e^{-x/2} dx.$$

If  $X$  is a variable distributed as  $\chi^2$  with  $\nu$  degrees of freedom,  $P/100$  is the probability that  $X \geq \chi^2_\nu(P)$ .

For  $\nu > 100$ ,  $\sqrt{2X}$  is approximately normally distributed with mean  $\sqrt{2\nu-1}$  and unit variance.

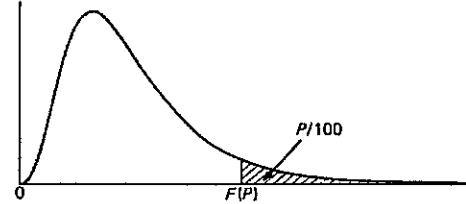


(The above shape applies for  $\nu \geq 3$  only. When  $\nu < 3$  the mode is at the origin.)

P	50	40	30	20	10	5	2.5	1	0.5	0.1	0.05
$\nu = 1$	0.4549	0.7083	1.074	1.642	2.706	3.841	5.024	6.635	7.879	10.83	12.12
2	1.386	1.833	2.408	3.219	4.605	5.991	7.378	9.210	10.60	13.82	15.20
3	2.366	2.946	3.665	4.642	6.251	7.815	9.348	11.34	12.84	16.27	17.73
4	3.357	4.045	4.878	5.989	7.779	9.488	11.14	13.28	14.86	18.47	20.00
5	4.351	5.132	6.064	7.289	9.236	11.07	12.83	15.09	16.75	20.52	22.11
6	5.348	6.211	7.231	8.558	10.64	12.59	14.45	16.81	18.55	22.46	24.10
7	6.346	7.283	8.383	9.803	12.02	14.07	16.01	18.48	20.28	24.32	26.02
8	7.344	8.351	9.524	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87
9	8.343	9.414	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67
10	9.342	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42
11	10.34	11.53	12.90	14.63	17.28	19.68	21.92	24.72	26.76	31.26	33.14
12	11.34	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82
13	12.34	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48
14	13.34	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11
15	14.34	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.72
16	15.34	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31
17	16.34	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88
18	17.34	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43
19	18.34	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97
20	19.34	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50
21	20.34	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01
22	21.34	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51
23	22.34	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00
24	23.34	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48
25	24.34	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95
26	25.34	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41
27	26.34	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.64	55.48	57.86
28	27.34	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30
29	28.34	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73
30	29.34	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16
32	31.34	33.38	35.66	38.47	42.58	46.19	49.48	53.49	56.33	62.49	65.00
34	33.34	35.44	37.80	40.68	44.90	48.60	51.97	56.06	58.96	65.25	67.80
36	35.34	37.50	39.92	42.88	47.21	51.00	54.44	58.62	61.58	67.99	70.59
38	37.34	39.56	42.05	45.08	49.51	53.38	56.90	61.16	64.18	70.70	73.35
40	39.34	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.09
50	49.33	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	86.66	89.56
60	59.33	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.7
70	69.33	72.36	75.69	79.71	85.53	90.53	95.02	100.4	104.2	112.3	115.6
80	79.33	82.57	86.12	90.41	96.58	101.9	106.6	112.3	116.3	124.8	128.3
90	89.33	92.76	96.52	101.1	107.6	113.1	118.1	124.1	128.3	137.2	140.8
100	99.33	102.9	106.9	111.7	118.5	124.3	129.6	135.8	140.2	149.4	153.2

TABLE 12(b). 5 PER CENT POINTS OF THE F-DISTRIBUTION

If  $F = \frac{X_1/X_2}{\nu_1/\nu_2}$ , where  $X_1$  and  $X_2$  are independent random variables distributed as  $\chi^2$  with  $\nu_1$  and  $\nu_2$  degrees of freedom respectively, then the probabilities that  $F \geq F(P)$  and that  $F \leq F(P)$  are both equal to  $P/100$ . Linear interpolation in  $\nu_1$  and  $\nu_2$  will generally be sufficiently accurate except when either  $\nu_1 > 12$  or  $\nu_2 > 40$ , when harmonic interpolation should be used.



(This shape applies only when  $\nu_1 \geq 3$ . When  $\nu_1 < 3$  the mode is at the origin.)

$\nu_1 =$	1	2	3	4	5	6	7	8	10	12	24	$\infty$
$\nu_2 = 1$	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	241.9	243.9	249.1	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.40	19.41	19.45	19.50
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.786	8.745	8.639	8.526
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.964	5.912	5.774	5.628
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.735	4.678	4.527	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.060	4.000	3.841	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.637	3.575	3.410	3.230
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.347	3.284	3.115	2.928
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.137	3.073	2.900	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	2.978	2.913	2.737	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.854	2.788	2.609	2.404
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.753	2.687	2.505	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.671	2.604	2.420	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.602	2.534	2.349	2.131
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.544	2.475	2.288	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.494	2.425	2.235	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.450	2.381	2.190	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.412	2.342	2.150	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.378	2.308	2.114	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.348	2.278	2.082	1.843
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.321	2.250	2.054	1.812
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.297	2.226	2.028	1.783
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.275	2.204	2.005	1.757
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.255	2.183	1.984	1.733
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.236	2.165	1.964	1.711
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.220	2.148	1.946	1.691
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.204	2.132	1.930	1.672
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.190	2.118	1.915	1.654
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.177	2.104	1.901	1.638
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.165	2.092	1.887	1.622
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244	2.142	2.070	1.864	1.594
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225	2.123	2.050	1.843	1.569
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209	2.106	2.033	1.824	1.547
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194	2.091	2.017	1.808	1.527
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.077	2.003	1.793	1.509
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	1.993	1.917	1.700	1.389
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.910	1.834	1.608	1.254
$\infty$	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.831	1.752	1.517	1.000